13
Vibrations and Waves

QUICK QUIZZES

1. Choice (d). To complete a full cycle of oscillation, the object must travel distance 2*A* to position $x = -A$ and then travel an additional distance 2*A* returning to the original position at $x = +A$.

2. Choice (c). The force producing harmonic oscillation is always directed toward the equilibrium position, and hence, directed opposite to the displacement from equilibrium. The acceleration is in the direction of the force. Thus, it is also always directed opposite to the displacement from equilibrium.

3. Choice (b). In simple harmonic motion, the force (and hence, the acceleration) is directly proportional to the displacement from equilibrium. Therefore, force and acceleration are both at a maximum when the displacement is a maximum.

4. Choice (a). The period of an object-spring system is $T = 2\pi\sqrt{m/k}$. Thus, increasing the mass by a factor of 4 will double the period of oscillation.

5. Choice (c). The total energy of the oscillating system is equal to $\frac{1}{2}kA^2$, where *A* is the amplitude of oscillation. Since the object starts from rest at displacement *A* in both cases, it has the same amplitude of oscillation in both cases.

6. Choice (d). The expressions for the total energy, maximum speed, and maximum acceleration are $E = \frac{1}{2}kA^2$, $v_{max} = A\sqrt{k/m}$, and $a_{max} = A(k/m)$, where *A* is the amplitude. Thus, all are changed by a change in amplitude. The period of oscillation is $T = 2\pi\sqrt{m/k}$ and is unchanged by altering the amplitude.

7. Choices (c) and (b). An accelerating elevator is equivalent to a gravitational field. Thus, if the elevator is accelerating upward, this is equivalent to an increased effective gravitational field magnitude *g*, and the period will decrease. Similarly, if the elevator is accelerating downward, the effective value of *g* is reduced and the period increases. If the elevator moves with constant velocity, the period of the pendulum is the same as that in the stationary elevator.

8. Choice (a). The clock will run *slow*. With a longer length, the period of the pendulum will increase. Thus, it will take longer to execute each swing, so that each second according to the clock will take longer than an actual second.

9. Choice (b). Greater. The value of *g* on the Moon is about one-sixth the value of *g* on Earth, so the period of the pendulum on the Moon will be greater than the period on Earth.

ANSWERS TO WARM-UP EXERCISES

1. (a) The maximum value of a sine function is +1, so that the maximum value of y(t) is

$$[y(t)]_{max} = (5.00\text{ m})[\sin(8.00\pi t)]_{max} = \boxed{5.00\text{ m}}$$

(b) The minimum value of a sine function is –1, so that the maximum value of y(t) is

$$[y(t)]_{min} = (5.00\text{ m})[\sin(8.00\pi t)]_{min} = \boxed{-5.00\text{ m}}$$

(c) The function has the form $y(t) = A\sin\omega t$, so $\omega = 8.00\pi$, and

$$T = \frac{2\pi}{\omega} == \frac{2\pi}{8\pi} = \boxed{0.250\text{ s}}$$

2. (a) From Equation 13.1, the magnitude of the spring force is given by

$$|F_s| = kx = (624\text{ N/m})(0.250\text{ m}) = \boxed{156\text{ N}}$$

(b) From Newton's second law in the horizontal direction

$$a = \frac{|F_s|}{m} = \frac{156\text{ N}}{4.00\text{ kg}} = \boxed{39.0\text{ m/s}^2}$$

3. Applying Newton's second law in the vertical direction, we obtain

$$\sum F_y = F_s - mg = ky - mg = 0 \quad \rightarrow \quad ky = mg$$

Solving for the distance y gives

$$y = \left(\frac{m}{k}\right)g = \left(\frac{7.20\text{ kg}}{575\text{ N/m}}\right)(9.80\text{ m/s}^2) = \boxed{0.123\text{ m}}$$

4. (a) The potential energy of the system is given by

$$PE_s = \frac{1}{2}kx^2 = \frac{1}{2}(75.0\text{ N/m})(0.200\text{ m})^2 = \boxed{1.50\text{ J}}$$

(b) We apply conservation of energy to the mass spring system:

$$\left(KE + PE_g + PE_s\right)_i = \left(KE + PE_g + PE_s\right)_f$$

Since the spring is horizontal, $\left(PE_g\right)_i = \left(PE_g\right)_f = 0.$ Taking the initial position when the mass is at maximum amplitude, $KE_i = 0,$ and taking the final position at the equilibrium point, $\left(PE_s\right)_f = 0.$ Thus,

$$0 + 0 + \frac{1}{2}kx^2 = \frac{1}{2}mv^2 + 0 + 0$$

which gives

$$v = \left(\sqrt{\frac{k}{m}}\right)x = \left(\sqrt{\frac{75.0 \text{ N/m}}{5.00 \text{ kg}}}\right)(0.200 \text{ m}) = \boxed{0.775 \text{ m/s}}$$

5. (a) The kinetic energy of the block is given by

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(4.00 \text{ kg})(7.00 \text{ m/s})^2 = \boxed{98.0 \text{ J}}$$

(b) We use conservation of energy for the block-spring system. Since the spring is horizontal, $\left(PE_g\right)_i = \left(PE_g\right)_f = 0.$ Taking the initial state to be that of the block prior to colliding with the spring, $\left(PE_s\right)_i = 0.$ We choose the final state of the system when the spring is at maximum compression, where $KE_f = 0.$ Then

$$\left(KE + PE_g + PE_s\right)_i = \left(KE + PE_g + PE_s\right)_f$$

gives

$$\frac{1}{2}mv^2 + 0 + 0 = 0 + 0 + \frac{1}{2}kx^2$$

and solving for x gives

$$x = \left(\sqrt{\frac{m}{k}}\right)v = \left(\sqrt{\frac{4.00 \text{ kg}}{1\,830 \text{ N/m}}}\right)(7.00 \text{ m/s}) = \boxed{0.327 \text{ m}}$$

6. (a) From Equation 13.11, the angular frequency of the block-spring system is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{628 \text{ N/m}}{8.00 \text{ kg}}} = \boxed{8.86 \text{ rad/s}}$$

(b) The frequency of oscillation is given by

$$f = \frac{\omega}{2\pi} = \frac{8.86 \text{ rad/s}}{2\pi} = \boxed{1.41 \text{ Hz}}$$

(c) From Equation, 13.9,

$$T = \frac{1}{f} = \frac{1}{1.41 \text{ Hz}} = \boxed{0.709 \text{ s}}$$

7. From Equation 13.14a, the position of a simple harmonic oscillator (SHO) is given by

$$x = A\cos(2\pi ft)$$

(a) The angular frequency is equal to $2\pi f,$ and is therefore

$$\omega = 2\pi f = 6.00\pi \text{ rad/s} = \boxed{18.8 \text{ rad/s}}$$

(b) The frequency of oscillation is given by

$$f = \frac{\omega}{2\pi} = \frac{6.00\pi \text{ rad/s}}{2\pi} = \boxed{3.00 \text{ Hz}}$$

(c) From Equation, 13.9,

$$T = \frac{1}{f} = \frac{1}{3.00 \text{ Hz}} = \boxed{0.333 \text{ s}}$$

(d) To find the spring constant, we note that $\omega = \sqrt{\frac{k}{m}},$ which gives

$$k = m\omega^2 = (5.00 \text{ kg})\left(18.8 \text{ rad/s}^2\right) = \boxed{1.78\times10^3 \text{ N/m}}$$

8. (a) The amplitude of oscillation is $\boxed{0.200 \text{ m}}$.

(b) The angular frequency of the oscillation is found from Equation 13.11:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{90.0 \text{ N/m}}{0.400 \text{ kg}}} = \boxed{15.0 \text{ rad/s}}$$

(c) The maximum speed of the mass is found from Equation 13.14b:

$$v_{\max} = A\omega = (0.200 \text{ m})\left(15.0 \text{ s}^{-1}\right) = \boxed{3.00 \text{ m/s}}$$

(d) The maximum acceleration of the mass is found from Equation 13.14c:

$$a_{\max} = A\omega^2 = (0.200 \text{ m})(15.0 \text{ s}^{-1})^2 = \boxed{45.0 \text{ m/s}^2}$$

9. (a) The frequency of a simple pendulum is

$$f = \frac{1}{T} = \frac{1}{2.50 \text{ s}} = \boxed{0.400 \text{ Hz}}$$

(b) The angular frequency is

$$\omega = 2\pi f = 2\pi(0.400 \text{ Hz}) = \boxed{2.51 \text{ rad/s}}$$

(c) To find the length of the pendulum, we note that

$$\omega = \sqrt{\frac{g}{L}}$$

Solving for L then yields

$$L = \frac{g}{\omega^2} = \frac{9.80 \text{ m/s}^2}{(2.51 \text{ rad/s})^2} = \boxed{1.55 \text{ m}}$$

10. (a) The wavelength of a wave is defined as the distance between successive crests, and in this case is $\boxed{4.00 \text{ m}}$.

(b) Successive crests arrive every 1.40 s, which is the period of the waves. Then,

$$f = \frac{1}{T} = \frac{1}{1.40 \text{ s}} = \boxed{0.714 \text{ Hz}}$$

(c) From Equation 13.17,

$$v = f\lambda = (0.714 \text{ Hz})(4.00 \text{ m}) = \boxed{2.86 \text{ m/s}}$$

11. (a) The linear mass density of the string is

$$\mu = \frac{m}{L} = \frac{0.005\ 00 \text{ kg}}{0.800 \text{ m}} = \boxed{0.006\ 25 \text{ kg/m}}$$

(b) From Equation 13.18,

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{275 \text{ N}}{0.006\ 25 \text{ kg/m}}} = \boxed{2.10 \times 10^2 \text{ m/s}}$$

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. Friction. This includes both air resistance and damping within the spring.

4. Each half-spring will have twice the spring constant of the full spring, as shown by the following argument. The force exerted by a spring is proportional to the separation of the coils as the spring is extended. Imagine that we extend a spring by a given distance and measure the distance between coils. We then cut the spring in half. If one of the half-springs is now extended by the same distance, the coils will be twice as far apart as they were for the complete spring. Thus, it takes twice as much force to stretch the half-spring, from which we conclude that the half-spring has a spring constant which is twice that of the complete spring.

6. No. The period of vibration is $T = 2\pi\sqrt{\ell/g}$ and g is smaller at high altitude. Therefore, the period is longer on the mountain top and the clock will run slower.

8. Shorten the pendulum to decrease the period between ticks.

10. The speed of the pulse is $v = \sqrt{F/\mu}$, so increasing the tension F in the hose increases the speed of the pulse. Filling the hose with water increases the mass per unit length μ, and will decrease the speed of the pulse.

12. As the temperature increases, the length of the pendulum will increase due to thermal expansion, and with a greater length, the period of the pendulum increases. Thus, it takes longer to execute each swing, so that each second according to the clock will take longer than an actual second. Consequently, the clock will run slow.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. 1.54 cm

4. **(a)** 1.1×10^2 N

(b) The graph should be a straight line passing through the origin with a positive slope of 1.0×10^3 N/m

6. **(a)** 327 N **(b)** 1.25×10^3 N

8. **(a)** 0.625 J **(b)** $0.791\ \text{m/s}$

10. **(a)** $575\ \text{N/m}$ **(b)** 46.0 J

12. $2.23\ \text{m/s}$

14. **(a)** $E = \frac{1}{2}kA^2$ **(b)** $\frac{1}{2}mv^2 = kx^2$ **(c)** $x = \pm A/\sqrt{3}$

16. **(a)** 4.58 N **(b)** 0.125 J **(c)** $18.3\ \text{m/s}^2$

(d) $1.00\ \text{m/s}$ **(e)** the speed would be lower

(f) The numeric value of the coefficient of kinetic friction would be required.

18. **(a)** 0.15 J **(b)** $0.78\ \text{m/s}$ **(c)** $18\ \text{m/s}^2$

20. $3.06\ \text{m/s}$

22. **(a)** $0.628\ \text{m/s}$ **(b)** 0.500 Hz **(c)** $3.14\ \text{rad/s}$

24. **(a)** 1.89 Hz **(b)** $33.7\ \text{N/m}$ **(c)** 0.118 m

26. **(a)** $3.9\times10^5\ \text{N/m}$ **(b)** 2.2 Hz

28. **(a)** 0.25 s **(b)** 4.0 Hz **(c)** 5.2 cm

(d) 21 ms

30. **(a)** $\pm A\sqrt{3}/2$ **(b)** $\pm A/\sqrt{2}$

32. **(a)** $250\ \text{N/m}$ **(b)** $22.4\ \text{rad/s}$, 3.56 Hz , 0.281 s

(c) 0.313 J **(d)** 5.00 cm **(e)** $1.12\ \text{m/s},\ 25.0\ \text{m/s}^2$

(f) 0.919 cm **(g)** $+1.10\ \text{m/s},\ -4.59\ \text{m/s}^2$

34. **(a)** 59.6 m **(b)** 37.5 s

36. 1.0015

38. $1.66\times10^{-2}\ \text{kg}\cdot\text{m}^2$

40. **(a)** 3.65 s **(b)** 6.41 s **(c)** 4.24 s

42. **(a)** 2.00 cm **(b)** 4.00 s **(c)** $\pi/2\ \text{rad/s}$

(d) $\pi\ \text{cm/s}$ **(e)** $4.93\ \text{cm/s}^2$ **(f)** $x = (2.00\ \text{cm})\sin(\pi t/2)$

44. **(a)** 0.357 Hz **(b)** $0.985\ \text{m/s}$

46. 5.72 mm

48. **(a)** 0.20 Hz **(b)** 0.25 Hz

50. 219 N

52. **(a)** The units of the first T are seconds, the units of the second are newtons.

(b) The first T is period of time; the second is force of tension.

54. $1.64\ \text{m/s}^2$

56. $7.07\ \text{m/s}$

58. $586\ \text{m/s}$

60. **(a)** $v = 2nL/t$ **(b)** $F = 4n^2ML/t^2$

62. **(a)** 0.25 m **(b)** $0.47\ \text{N/m}$ **(c)** 0.23 m

(d) $0.12\ \text{m/s}$

64. **(a)** 5.10 ms **(b)** 1.75 m

66. 0.990 m

68. **(a)** $100\ \text{m/s}$ **(b)** 374 J

70. **(a)** $19.8\ \text{m/s}$ **(b)** 8.95 m

72. **(a)** and **(b)** See Solution for proofs.

74. 1.3 cm/s

76. **(a)** $\Sigma F_y = -ky_f - mg = mv^2/(L - y_f)$

(b) $mv^2 = 2mg\left(L - y_f\right) - ky_f^2$ **(c)** $y_f = -0.110\ \text{m}$ **(d)** greater than

PROBLEM SOLUTIONS

13.1 (a) Taking to the right as positive, the spring force acting on the block at the instant of release is

$$F_s = -kA = -\left(130\ \text{N/m}\right)\left(+0.13\ \text{m}\right) = -17\ \text{N} \quad \text{or} \quad \boxed{17\ \text{N to the left}}$$

(b) At this instant, the acceleration is

$$a = \frac{F_s}{m} = \frac{-17\ \text{N}}{0.60\ \text{kg}} = -28\ \text{m/s}^2 \qquad \text{or} \qquad \boxed{a = 28\ \text{m/s}^2\ \text{to the left}}$$

13.2 The force compressing the spring is the weight of the object. Thus, the spring will be compressed a distance of

$$|x| = \frac{|F|}{k} = \frac{mg}{k} = \frac{\left(2.30\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{1.46 \times 10^3\ \text{N/m}} = 1.54 \times 10^{-2}\ \text{m} = \boxed{1.54\ \text{cm}}$$

13.3 Assuming the spring obeys Hooke's law, the magnitude of the force required to displace the end a distance $|x|$ from the equilibrium position (by either compressing or stretching the spring) is $|F| = k|x|$ where k is the force constant of the spring.

(a) If If $x = -4.80\ \text{cm}$, the required force is $|F| = k|x| = \left(137\ \text{N/m}\right)\left(4.80 \times 10^{-2}\ \text{m}\right) = \boxed{6.58\ \text{N}}$

(b) If $x = +7.36\ \text{cm}$, the required force is $|F| = k|x| = \left(137\ \text{N/m}\right)\left(7.36 \times 10^{-2}\ \text{m}\right) = \boxed{10.1\ \text{N}}$

13.4 (a) The spring constant is $k = \dfrac{|F_s|}{x} = \dfrac{mg}{x} = \dfrac{50\ \text{N}}{5.0 \times 10^{-2}\ \text{m}} = 1.0 \times 10^3\ \text{N/m}$

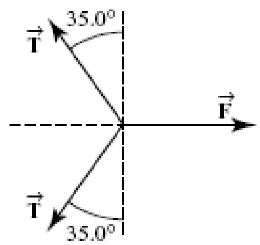
$$F = |F_s| = kx = \left(1.0 \times 10^3\ \text{N/m}\right)\left(0.11\ \text{m}\right) = \boxed{1.1 \times 10^2\ \text{N}}$$

(b) The graph will be a $\boxed{\text{straight line passing through the origin}}$ with a slope equal to $k = 1.0 \times 10^3\ \text{N/m}$.

13.5 When the system is in equilibrium, the tension in the spring $F = k|x|$ must equal the weight of the object. Thus,

$$k|x| = mg \text{ giving } m = \frac{k|x|}{g} = \frac{(47.5 \text{ N/m})(5.00\times10^{-2} \text{ m})}{9.80 \text{ m/s}^2} = \boxed{0.242 \text{ kg}}$$

13.6 (a) The free-body diagram of the point in the center of the string is given at the right. From this, we see that



$$\Sigma F_x = 0 \Rightarrow F - 2T\sin 35.0° = 0$$

or $$T = \frac{F}{2\sin 35.0°} = \frac{375 \text{ N}}{2\sin 35.0°} = \boxed{327 \text{ N}}$$

(b) Since the bow requires an applied horizontal force of 375 N to hold the string at 35.0° from the vertical, the tension in the spring must be 375 N when the spring is stretched 30.0 cm. Thus, the spring constant is

$$k = \frac{F}{x} = \frac{375 \text{ N}}{0.300 \text{ m}} = \boxed{1.25\times10^3 \text{ N/m}}$$

13.7 (a) When the block comes to equilibrium, $\Sigma F_y = -ky_0 - mg = 0$ giving

$$y_0 = -\frac{mg}{k} = -\frac{(10.0 \text{ kg})(9.80 \text{ m/s}^2)}{475 \text{ N/m}} = -0.206 \text{ m}$$

or the equilibrium position is $\boxed{0.206 \text{ m}}$ *below* the unstretched position of the lower end of the spring.

(b) When the elevator (and everything in it) has an upward acceleration of $a = 2.00 \text{ m/s}^2$, applying Newton's second law to the block gives

$$\Sigma F_y = -k(y_0 + y) - mg = ma_y \quad \text{or} \quad \Sigma F_y = (-ky_0 - mg) - ky = ma_y$$

where $y = 0$ at the equilibrium position of the block. Since $-ky_0 - mg = 0$ [see part (a)], this becomes $-ky = ma$ and the new position of the block is

$$y = \frac{ma_y}{-k} = \frac{(10.0 \text{ kg})(+2.00 \text{ m/s}^2)}{-475 \text{ N/m}} = -4.21\times10^{-2} \text{ m} = \boxed{-4.21 \text{ cm}}$$

or $\boxed{4.21 \text{ cm below the equilibrium position}}$.

(c) When the cable breaks, the elevator and its contents will be in free-fall with $a_y = -g$. The new "equilibrium" position of the block is found from $\Sigma F_y = -ky_0' - mg = m(-g)$, which yields $y_0' = 0$. When the cable snapped, the block was at rest relative to the elevator at distance $y_0 + y = 0.206 \text{ m} + 0.042\,1 \text{ m} = 0.248 \text{ m}$ below the new "equilibrium" position. Thus, while the elevator is in free-fall, the block will oscillate with $\boxed{\text{amplitude} = 0.248 \text{ m}}$ about the new "equilibrium" position, which is the unstretched position of the spring's lower end.

13.8 (a) The work required to stretch the spring equals the elastic potential energy of the spring in the stretched condition, or

$$W = \frac{1}{2}kx^2 = \frac{1}{2}\left(5.00\times10^2\ \text{N/m}\right)\left(5.00\times10^{-2}\ \text{m}\right)^2 = \boxed{0.625\ \text{J}}$$

(b) In the initial condition, the spring-block system is at rest $(KE_i = 0)$ with elastic potential energy of $PE_{s,i} = 0.625$ J. Since the spring force is conservative, conservation of energy gives $KE_f + PE_{s,f} = KE_i + PE_{s,i} = 0.625$ J. Thus, when the block is at the equilibrium position $(PE_{s,f} = 0)$, we have $KE_f = \frac{1}{2}mv_f^2 = 0.625$ J, or

$$v_f = \sqrt{\frac{2(0.625\ \text{J})}{m}} = \sqrt{\frac{2(0.625\ \text{J})}{2.00\ \text{kg}}} = \boxed{0.791\ \text{m/s}}$$

13.9 (a) Assume the rubber bands obey Hooke's law. Then, the force constant of each band is

$$k = \frac{F_s}{x} = \frac{15\ \text{N}}{1.0\times10^{-2}\ \text{m}} = 1.5\times10^3\ \text{N/m}$$

Thus, when both bands are stretched 0.20 m, the total elastic potential energy is

$$PE_s = 2\left(\frac{1}{2}kx^2\right) = \left(1.5\times10^3\ \text{N/m}\right)(0.20\ \text{m})^2 = \boxed{60\ \text{J}}$$

(b) Conservation of mechanical energy gives $\left(KE + PE_s\right)_f = \left(KE + PE_s\right)_i$, or

$$\frac{1}{2}mv^2 + 0 = 0 + 60\ \text{J} \qquad \text{so} \qquad v = \sqrt{\frac{2(60\ \text{J})}{50\times10^{-3}\ \text{kg}}} = \boxed{49\ \text{m/s}}$$

13.10 (a) $k = \dfrac{F_{max}}{x_{max}} = \dfrac{230\ \text{N}}{0.400\ \text{m}} = \boxed{575\ \text{N/m}}$

(b) $\textit{work done} = PE_s = \dfrac{1}{2}kx^2 = \dfrac{1}{2}(575\ \text{N/m})(0.400)^2 = \boxed{46.0\ \text{J}}$

13.11 From conservation of mechanical energy,

$$\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i \ \text{ or } \ 0 + mgh_f + 0 = 0 + 0 + \frac{1}{2}kx_i^2,$$

giving

$$k = \frac{2mgh_f}{x_i^2} = \frac{2(0.100\ \text{kg})\left(9.80\ \text{m/s}^2\right)(0.600\ \text{m})}{\left(2.00\times10^{-2}\ \text{m}\right)^2} = \boxed{2.94\times10^3\ \text{N/m}}$$

13.12 Conservation of mechanical energy, $\left(KE + PE_g + PE_s\right)_i = \left(KE + PE_g + PE_s\right)_f$, gives

$$\frac{1}{2}mv_i^2 + 0 + 0 = 0 + 0 + \frac{1}{2}kx_f^2,$$

or $v_i = \sqrt{\dfrac{k}{m}}\,x_i = \sqrt{\dfrac{5.00\times10^6\ \text{N/m}}{1000\ \text{kg}}}\left(3.16\times10^{-2}\ \text{m}\right) = \boxed{2.23\ \text{m/s}}$

13.13 An unknown quantity of mechanical energy is converted into internal energy during the collision. Thus, we apply conservation of momentum from just before to just after the collision and obtain $mv_i + M(0) = (M+m)V$, or the speed of the block and embedded bullet just after collision is $V = (m/M+m)v_i$. We now use conservation of mechanical energy, $(KE + PE_s)_f = (KE + PE_s)_i$, from just after collision until the block comes to rest. This gives $0 + \frac{1}{2}kx_f^2 = \frac{1}{2}(M+m)V^2 + 0$, or

$$x_f = V\sqrt{\frac{M+m}{k}} = v_i\left(\frac{m}{M+m}\right)\sqrt{\frac{M+m}{k}} = \frac{mv_i}{\sqrt{(M+m)k}}$$

yielding $$x_f = \frac{(10.0\times10^{-3}\text{ kg})(300\text{ m/s})}{\sqrt{(2.01\text{ kg})(19.6\text{ N/m})}} = \boxed{0.478\text{ m}}$$

13.14 (a) At either of the turning points, $x = \pm A$, the constant total energy of the system is momentarily stored as elastic potential energy in the spring. Thus, $\boxed{E = \frac{1}{2}kA^2}$.

(b) When the object is distance x from the equilibrium position, the elastic potential energy is $PE_s = kx^2/2$ and the kinetic energy is $KE = mv^2/2$ At the position where $KE = 2PE_s$, it is necessary that

$$\frac{1}{2}mv^2 = 2\left(\frac{1}{2}kx^2\right) \quad \text{or} \quad \boxed{\frac{1}{2}mv^2 = kx^2}$$

(c) When $KE = 2PE_s$, conservation of energy gives $E = KE + PE_s = 2(PE_s) + PE_s = 3PE_s$, or

$$\frac{1}{2}kA^2 = 3\left(\frac{1}{2}kx^2\right) \quad \Rightarrow \quad x = \pm\sqrt{\frac{\cancel{k}A^2/\cancel{2}}{3\cancel{k}/\cancel{2}}} \quad \text{or} \quad \boxed{x = \pm\frac{A}{\sqrt{3}}}$$

13.15 (a) At maximum displacement from equilibrium, all of the energy is in the form of elastic potential energy, giving $E = \frac{1}{2}kx_{max}^2$, and

$$k = \frac{2E}{x_{max}^2} = \frac{2(47.0\text{ J})}{(0.240\text{ m})^2} = \boxed{1.63\times10^3\text{ N/m}}$$

(b) At the equilibrium position $(x = 0)$, the spring is momentarily in its relaxed state and $PE_s = 0$, so all of the energy is in the form of kinetic energy. This gives

$$KE|_{x=0} = \frac{1}{2}mv_{max}^2 = E = \boxed{47.0\text{ J}}$$

(c) If, at the equilibrium position, $v = v_{max} = 3.45\text{ m/s}$, the mass of the block is

$$m = \frac{2E}{v_{max}^2} = \frac{2(47.0\text{ J})}{(3.45\text{ m/s})^2} = \boxed{7.90\text{ kg}}$$

(d) At any position, the constant total energy is $E = KE + PE_s = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$, so at $x = 0.160\text{ m}$

$$v = \sqrt{\frac{2E - kx^2}{m}} = \sqrt{\frac{2(47.0\text{ J}) - (1.63\times10^3\text{ N/m})(0.160\text{ m})^2}{7.90\text{ kg}}} = \boxed{2.57\text{ m/s}}$$

(e) At $x = 0.160\text{ m}$, where $v = 2.57\text{ m/s}$, the kinetic energy is

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(7.90\text{ kg})(2.57\text{ m/s})^2 = \boxed{26.1\text{ J}}$$

(f) At $x = 0.160$ m, where $KE = 26.1$ J, the elastic potential energy is

$$PE_s = E - KE = 47.0\text{ J} - 26.1\text{ J} = \boxed{20.9\text{ J}}$$

or alternately: $$PE_s = \frac{1}{2}kx^2 = \frac{1}{2}\left(1.63\times10^3\text{ N/m}\right)(0.160\text{ m})^2 = \boxed{20.9\text{ J}}$$

(g) At the first turning point (for which $x < 0$ since the block started from rest at $x = +0.240$ m and has passed througsh the equilibrium at $x = 0$), all of the remaining energy is in the form of elastic potential energy, so

$$\frac{1}{2}kx^2 = E - E_{lost} = 47.0\text{ J} - 14.0\text{ J} = 33.0\text{ J}$$

and $$x = -\sqrt{\frac{2(33.0\text{ J})}{k}} = -\sqrt{\frac{2(33.0\text{ J})}{1.63\times10^3\text{ N/m}}} = \boxed{-0.201\text{ m}}$$

13.16 (a) $$F = k|x| = (83.8\text{ N/m})\left(5.46\times10^{-2}\text{ m}\right) = \boxed{4.58\text{ N}}$$

(b) $$E = PE_s = \frac{1}{2}kx^2 = \frac{1}{2}(83.8\text{ N/m})\left(5.46\times10^{-2}\text{ m}\right)^2 = \boxed{0.125\text{ J}}$$

(c) While the block was held stationary at $x = 5.46$ cm, $\Sigma F_x = -F_s + F = 0$, or the spring force was equal in magnitude and oppositely directed to the applied force. When the applied force is suddenly removed, there is a net force $F_s = 4.58$ N directed toward the equilibrium position acting on the block. This gives the block an acceleration having magnitude

$$|a| = \frac{F_s}{m} = \frac{4.58\text{ N}}{0.250\text{ kg}} = \boxed{18.3\text{ m/s}^2}$$

(d) At the equilibrium position, $PE_s = 0$, so the block has kinetic energy $KE = E = 0.125$ J and speed

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(0.125\text{ J})}{0.250\text{ kg}}} = \boxed{1.00\text{ m/s}}$$

(e) If the surface was rough, the block would spend energy overcoming a retarding friction force as it moved toward the equilibrium position, causing it to arrive at that position with $\boxed{\text{a lower speed}}$ than that computed above.

(f) Computing a numeric value for this lower speed requires knowledge of the $\boxed{\text{coefficient of kinetic friction}}$ between the block and surface.

13.17 From conservation of mechanical energy, $\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i$,we have $\frac{1}{2}mv^2 + 0 + \frac{1}{2}kx^2 = 0 + 0 + \frac{1}{2}kA^2$, or

$$v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$$

(a) The speed is a maximum at the equilibrium position, $x = 0$.

$$v_{max} = \sqrt{\frac{k}{m}A^2} = \sqrt{\frac{(19.6\text{ N/m})}{(0.40\text{ kg})}(0.040\text{ m})^2} = \boxed{0.28\text{ m/s}}$$

(b) When $x = -0.015$ m,

$$v = \sqrt{\frac{(19.6\text{ N/m})}{(0.40\text{ kg})}\left[(0.040\text{ m})^2 - (-0.015\text{ m})^2\right]} = \boxed{0.26\text{ m/s}}$$

(c) When $x = +0.015$ m,

$$v = \sqrt{\frac{(19.6 \text{ N/m})}{(0.40 \text{ kg})}\left[(0.040 \text{ m})^2 - (+0.015 \text{ m})^2\right]} = \boxed{0.26 \text{ m/s}}$$

(d) If $v = \frac{1}{2}v_{max}$, then

$$\sqrt{\frac{k}{m}(A^2 - x^2)} = \frac{1}{2}\sqrt{\frac{k}{m}A^2}$$

This gives $A^2 - x^2 = \frac{1}{4}A^2$, or

$$x = A\frac{\sqrt{3}}{2} = (4.0 \text{ cm})\frac{\sqrt{3}}{2} = \boxed{3.5 \text{ cm}}$$

13.18 (a) $KE = 0$ at $x = A$, so $E = KE + PE_s = 0 + \frac{1}{2}kA^2$, or the total energy is

$$E = \frac{1}{2}kA^2 = \frac{1}{2}(250 \text{ N/m})(0.035 \text{ m})^2 = \boxed{0.15 \text{ J}}$$

(b) The maximum speed occurs at the equilibrium position where $PE_s = 0$. Thus, $E = \frac{1}{2}mv_{max}^2$, or

$$v_{max} = \sqrt{\frac{2E}{m}} = A\sqrt{\frac{k}{m}} = (0.035 \text{ m})\sqrt{\frac{250 \text{ N/m}}{0.50 \text{ kg}}} = \boxed{0.78 \text{ m/s}}$$

(c) The acceleration is $a = \Sigma F/m = -kx/m$. Thus, $a = a_{max}$ at $x = -x_{max} = -A$.

$$a_{max} = \frac{-k(-A)}{m} = \left(\frac{k}{m}\right)A = \left(\frac{250 \text{ N/m}}{0.50 \text{ kg}}\right)(0.035 \text{ m}) = \boxed{18 \text{ m/s}^2}$$

13.19 The maximum speed occurs at the equilibrium position and is $v_{max} = A\sqrt{k/m}$. Thus,

$$m = \frac{kA^2}{v_{max}^2} = \frac{(16.0 \text{ N/m})(0.200 \text{ m})^2}{(0.400 \text{ m/s})^2} = 4.00 \text{ kg}$$

and

$$F_g = mg = (4.00 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{39.2 \text{ N}}$$

13.20 $$v = \sqrt{\frac{k}{m}(A^2 - x^2)} = \sqrt{\left(\frac{10.0 \text{ N/m}}{50.0 \times 10^{-3} \text{ kg}}\right)\left[(0.250 \text{ m})^2 - (0.125 \text{ m})^2\right]} = \boxed{3.06 \text{ m/s}}$$

13.21 (a) (a) $PE_{si} = \frac{1}{2}kx_i^2 = \frac{1}{2}(850 \text{ N/m})(6.00 \times 10^{-2} \text{ m})^2 = \boxed{1.53 \text{ J}}$

(b) Since the surface is frictionless, the total energy of the block-spring system is constant. Thus, $KE + PE_s = KE_i + PE_{si} = 0 + 1.53$ J. At the equilibrium position, $PE_s = 0$, so the kinetic energy must be $KE_{max} = \frac{1}{2}mv_{max}^2 = 1.53$ J, which yields

$$v_{max} = \sqrt{\frac{2KE_{max}}{m}} = \sqrt{\frac{2(1.53 \text{ J})}{1.00 \text{ kg}}} = \boxed{1.75 \text{ m/s}}$$

(c) At $x = x_i/2 = 3.00$ cm, the elastic potential energy is $PE_s = \frac{1}{2}kx^2$, and the conservation of energy gives $KE + PE_s = E$, or $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = E$ and

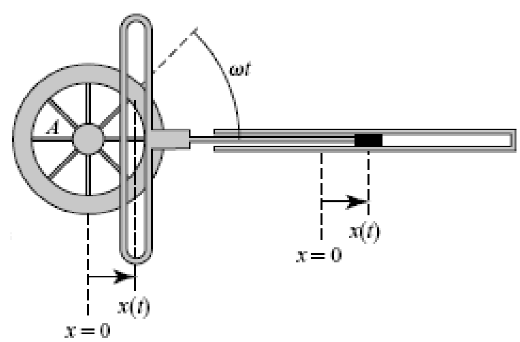
$$v = \sqrt{\frac{2E - kx^2}{m}} = \sqrt{\frac{2(1.53\text{ J}) - (850\text{ N/m})(3.00\times10^{-2}\text{ m})^2}{1.00\text{ kg}}} = \boxed{1.51\text{ m/s}}$$

13.22 (a) $v_t = \frac{2\pi r}{T} = \frac{2\pi(0.200\text{ m})}{2.00\text{ s}} = \boxed{0.628\text{ m/s}}$

(b) $f = \frac{1}{T} = \frac{1}{2.00\text{ s}} = \boxed{0.500\text{ Hz}}$

(c) $\omega = \frac{2\pi}{T} = \frac{2\pi}{2.00\text{ s}} = \boxed{3.14\text{ rad/s}}$

13.23 The angle of the crank pin is $\theta = \omega t$. Its x-coordinate is $x = A\cos\theta = A\cos\omega t$, where A is the distance from the center of the wheel to the crank pin. The displacement of the piston from its zero position (i.e., its location when $\theta = \omega t = \pi/2$) is the same as that of the crankpin, $x(t) = A\cos\omega t$. This is of the correct form to describe simple harmonic motion. Hence, one must conclude that the motion is indeed simple harmonic.



13.24 (a) (a) $f = \frac{1}{T} = \frac{1}{0.528\text{ s}} = \boxed{1.89\text{ Hz}}$

(b) The period of oscillation of an object-spring system is $T = 2\pi\sqrt{m/k}$, so the force constant is

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(0.238\text{ kg})}{(0.528\text{ s})^2} = \boxed{33.7\text{ N/m}}$$

(c) At the turning points $(x = \pm A)$ in the oscillation, all of the energy is temporarily stored as elastic potential energy, or $E = kA^2/2$. Thus,

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(0.234\text{ J})}{33.7\text{ N/m}}} = \boxed{0.118\text{ m}}$$

13.25 The spring constant is found from

$$k = \frac{F_s}{x} = \frac{mg}{x} = \frac{(0.010\text{ kg})(9.80\text{ m/s}^2)}{3.9\times10^{-2}\text{ m}} = 2.5\text{ N/m}$$

When the object attached to the spring has mass $m = 25$ g, the period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.025\text{ kg}}{2.5\text{ N/m}}} = \boxed{0.63\text{ s}}$$

13.26 (a) The springs compress 0.80 cm when supporting an additional load of $m = 320$ kg. Thus, the spring constant is

$$k = \frac{\Delta F_s}{\Delta x} = \frac{mg}{\Delta x} = \frac{(320\text{ kg})(9.80\text{ m/s}^2)}{0.80\times 10^{-2}\text{ m}} = \boxed{3.9\times 10^5\text{ N/m}}$$

(b) When the empty car, $M = 2.0\times 10^3$ kg, oscillates on the springs, the frequency will

be be $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = \frac{1}{2\pi}\sqrt{\frac{3.9\times 10^5\text{ N/m}}{2.0\times 10^3\text{ kg}}} = \boxed{2.2\text{ Hz}}$

13.27 (a) The period of oscillation is $T = 2\pi\sqrt{m/k}$, where *k* is the spring constant and *m* is the mass of the object attached to the end of the spring. Hence,

$$T = 2\pi\sqrt{\frac{0.250\text{ kg}}{9.5\text{ N/m}}} = \boxed{1.0\text{ s}}$$

(b) If the cart is released from rest when it is 4.5 cm from the equilibrium position, the amplitude of oscillation will be $A = 4.5\text{ cm} = 4.5\times 10^{-2}\text{ m}$. The maximum speed is then given by

$$v_{max} = A\omega = A\sqrt{\frac{k}{m}} = \left(4.5\times 10^{-2}\text{ m}\right)\sqrt{\frac{9.5\text{ N/m}}{0.250\text{ kg}}} = \boxed{0.28\text{ m/s}}$$

(c) When the cart has a displacement of $x = 2.0$ cm from the equilibrium position, its speed will be

$$v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = \sqrt{\frac{9.5\text{ N/m}}{0.250\text{ kg}}\left[(0.045\text{ m})^2 - (0.020\text{ m})^2\right]} = \boxed{0.25\text{ m/s}}$$

13.28 The general expression for the position as a function of time for an object undergoing simple harmonic motion with $x = 0$ at $t = 0$ is $x = A\sin(\omega t)$. Thus, if $x = (5.2\text{ cm})\sin(8.0\pi\cdot t)$, we have that the amplitude is $A = 5.2$ cm and the angular frequency is $\omega = 8.0\pi$ rad/s.

(a) The period is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{8.0\pi\text{ s}^{-1}} = \boxed{0.25\text{ s}}$$

(b) The frequency of motion is

$$f = \frac{1}{T} = \frac{1}{0.25\text{ s}} = 4.0\text{ s}^{-1} = \boxed{4.0\text{ Hz}}$$

(c) As discussed above, the amplitude of the motion is $\boxed{A = 5.2\text{ cm}}$.

(d) **Note:** For this part, your calculator should be set to operate in *radians mode*. If $x = 2.6$ cm, then

$$\omega t = \sin^{-1}\left(\frac{x}{A}\right) = \sin^{-1}\left(\frac{2.6\text{ cm}}{5.2\text{ cm}}\right) = \sin^{-1}(0.50) = 0.52\text{ radians}$$

and

$$t = \frac{0.52\text{ rad}}{\omega} = \frac{0.52\text{ rad}}{8.0\pi\text{ rad/s}} = 2.1\times 10^{-2}\text{ s} = 21\times 10^{-3}\text{ s} = \boxed{21\text{ ms}}$$

13.29 (a) At the equilibrium position, the total energy of the system is in the form of kinetic energy and $\frac{1}{2}mv_{max}^2 = E$, so the maximum speed is

$$v_{max} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(5.83\ \text{J})}{0.326\ \text{kg}}} = \boxed{5.98\ \text{m/s}}$$

(b) The period of an object-spring system is $T = 2\pi\sqrt{m/k}$, so the force constant of the spring is

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(0.326\ \text{kg})}{(0.250\ \text{s})^2} = \boxed{206\ \text{N/m}}$$

(c) At the turning points, $x = \pm A$, the total energy of the system is in the form of elastic potential energy, or $E = \frac{1}{2}kA^2$, giving the amplitude as

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(5.83\ \text{J})}{206\ \text{N/m}}} = \boxed{0.238\ \text{m}}$$

13.30 For a system executing simple harmonic motion, the total energy may be written as $E = KE + PE_s = \frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$, where A is the amplitude and v_{max} is the speed at the

equilibrium position. Observe from this expression, that we may write $v_{max}^2 = kA^2/m$.

(a) If $v = \frac{1}{2}v_{max}$, then $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}mv_{max}^2$ becomes

$$\frac{1}{2}m\left(\frac{v_{max}^2}{4}\right) + \frac{1}{2}kx^2 = \frac{1}{2}mv_{max}^2$$

and gives

$$x^2 = \frac{3}{4}\left(\frac{m}{k}\right)v_{max}^2 = \frac{3}{4}\left(\frac{m}{k}\right)\left[\frac{k}{m}A^2\right] = \frac{3A^2}{4} \quad \text{or} \quad \boxed{x = \pm\frac{A\sqrt{3}}{2}}$$

(b) If the elastic potential energy is $PE_s = \frac{1}{2}E$, we have

$$\frac{1}{2}kx^2 = \frac{1}{2}\left(\frac{1}{2}kA^2\right) \quad \text{or} \quad x^2 = \frac{A^2}{2} \quad \text{and} \quad \boxed{x = \pm\frac{A}{\sqrt{2}}}$$

13.31 **Note**: Your calculator must be in radians mode for part (a) of this problem.

(a) The angular frequency of this oscillation is $\omega = \sqrt{k/m}$ and the displacement at time t is $x = A\cos\omega t$. At $t = 3.50$ s, the spring force will be $F = -kx = -kA\cos(\omega t)$, or

$$F = -\left(5.00\ \frac{\text{N}}{\text{m}}\right)(3.00\ \text{m})\cos\left[\left(\sqrt{\frac{5.00\ \text{N/m}}{2.00\ \text{kg}}}\right)(3.50\ \text{s})\right] = -11.0\ \text{N},$$

or $F = \boxed{11.0\ \text{N directed to the left}}$

(b) The time required for one complete oscillation is $T = 2\pi/\omega = 2\pi\sqrt{m/k}$. Hence, the number of oscillations made in 3.50 s is

$$N = \frac{\Delta t}{T} = \frac{\Delta t}{2\pi\sqrt{m/k}} = \frac{3.50\ \text{s}}{2\pi}\sqrt{\frac{5.00\ \text{N/m}}{2.00\ \text{kg}}} = \boxed{0.881}$$

13.32 (a) $k = \frac{F}{x} = \frac{7.50 \text{ N}}{3.00 \times 10^{-2} \text{ m}} = \boxed{250 \text{ N/m}}$

(b) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{250 \text{ N/m}}{0.500 \text{ kg}}} = \boxed{22.4 \text{ rad/s}}$,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{250 \text{ N/m}}{0.500 \text{ kg}}} = \boxed{3.56 \text{ Hz}},$$

and $T = \frac{1}{f} = \frac{1}{3.56 \text{ Hz}} = \boxed{0.281 \text{ s}}$

(c) At $t = 0$, $t = 0$, $v = 0$ and $x = 5.00 \times 10^{-2}$, so the total energy of the oscillator is

$$E = KE + PE_s = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = 0 + \frac{1}{2}(250 \text{ N/m})(5.00 \times 10^{-2} \text{ m})^2 = \boxed{0.313 \text{ J}}$$

(d) When $x = A$, $v = 0$ so $E = KE + PE_s = 0 + \frac{1}{2}kA^2$.

Thus, $A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(0.313 \text{ J})}{250 \text{ N/m}}} = 5.00 \times 10^{-2} \text{ m} = \boxed{5.00 \text{ cm}}$

(e) At $x = 0$, $KE = \frac{1}{2}mv_{max}^2 = E$, or $v_{max} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(0.313 \text{ J})}{0.500 \text{ kg}}} = \boxed{1.12 \text{ m/s}}$

$$a_{max} = \frac{F_{max}}{m} = \frac{kA}{m} = \frac{(250 \text{ N/m})(5.00 \times 10^{-2} \text{ m})}{0.500 \text{ kg}} = \boxed{25.0 \text{ m/s}^2}$$

Note: To solve parts (f) and (g), your calculator should be set in *radians mode*.

(f) At $t = 0.500$ s, Equation 13.14a gives the displacement as

$$x = A\cos(\omega t) = A\cos\left(t\sqrt{k/m}\right) = (5.00 \text{ cm})\cos\left[(0.500 \text{ s})\sqrt{\frac{250 \text{ N/m}}{0.500 \text{ kg}}}\right] = \boxed{0.919 \text{ cm}}$$

(g) From Equation 13.14b, the velocity at $t = 0.500$ s is

$$v = -A\omega\sin(\omega t) = -A\sqrt{k/m}\sin\left(t\sqrt{k/m}\right)$$
$$= -\left(5.00 \times 10^{-2} \text{ m}\right)\sqrt{\frac{250 \text{ N/m}}{0.500 \text{ kg}}}\sin\left[(0.500 \text{ s})\sqrt{\frac{250 \text{ N/m}}{0.500 \text{ kg}}}\right] = \boxed{+1.10 \text{ m/s}}$$

and from Equation 13.14c, the acceleration at this time is

$$a = -A\omega^2\cos(\omega t) = -A(k/m)\cos\left(t\sqrt{k/m}\right)$$
$$= -\left(5.00 \times 10^{-2} \text{ m}\right)\left(\frac{250 \text{ N/m}}{0.500 \text{ kg}}\right)\cos\left[(0.500 \text{ s})\sqrt{\frac{250 \text{ N/m}}{0.500 \text{ kg}}}\right] = \boxed{-4.59 \text{ m/s}^2}$$

13.33 From Equation 13.6, $v = \pm\sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = \pm\sqrt{\omega^2\left(A^2 - x^2\right)}$

Hence, $v = \pm\omega\sqrt{A^2 - A^2\cos^2(\omega t)} = \pm\omega A\sqrt{1-\cos^2(\omega t)} = \boxed{\pm\omega A\sin(\omega t)}$

From Equation 13.2, $a = -\frac{k}{m}x = -\omega^2\left[A\cos(\omega t)\right] = \boxed{-\omega^2 A\cos(\omega t)}$

13.34 (a) The height of the tower is almost the same as the length of the pendulum. From $T = 2\pi\sqrt{L/g}$, we obtain

$$L = \frac{gT^2}{4\pi^2} = \frac{\left(9.80\ \text{m/s}^2\right)(15.5\ \text{s})^2}{4\pi^2} = \boxed{59.6\ \text{m}}$$

(b) On the Moon, where $g = 1.67\ \text{m/s}^2$, the period will be

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{59.6\ \text{m}}{1.67\ \text{m/s}^2}} = \boxed{37.5\ \text{s}}$$

13.35 (a) The period is the time for one complete oscillation. Hence,

$$T = \frac{2.00\ \text{min}}{82}\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = \frac{120\ \text{s}}{82.0} \qquad \text{or} \qquad T = \boxed{1.46\ \text{s}}$$

(b) The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\ell/g}$, so the local acceleration of gravity must be

$$g = \frac{4\pi^2\ell}{T^2} = \frac{4\pi^2(0.520\ \text{m})}{(120\ \text{s}/82.0)^2} = \boxed{9.59\ \text{m/s}^2}$$

13.36 The period in Tokyo is $T_T = 2\pi\sqrt{\frac{L_T}{g_T}}$ and the period in Cambridge is $T_C = 2\pi\sqrt{\frac{L_C}{g_C}}$.

We know that $T_T = T_C = 2.000\ \text{s}$, from which we see that

$$\frac{L_T}{g_T} = \frac{L_C}{g_C} \qquad \text{or} \qquad \frac{g_C}{g_T} = \frac{L_C}{L_T} = \frac{0.994\,2}{0.992\,7} = \boxed{1.001\,5}$$

13.37 (a) The period of the pendulum is $T = 2\pi\sqrt{L/g}$. Thus, on the Moon where the free-fall acceleration is smaller, the period will be longer and the clock will run $\boxed{\text{slow}}$.

(b) The ratio of the pendulum's period on the Moon to that on Earth is

$$\frac{T_{\text{Moon}}}{T_{\text{Earth}}} = \frac{2\pi\sqrt{L/g_{\text{Moon}}}}{2\pi\sqrt{L/g_{\text{Earth}}}} = \sqrt{\frac{g_{\text{Earth}}}{g_{\text{Moon}}}}$$

Hence, the pendulum of the clock on Earth makes $\sqrt{g_{\text{Earth}}/g_{\text{Moon}}}$ "ticks" while the clock on the Moon is making 1.00 "tick." After the Earth clock has ticked off 24.0 h and again reads 12:00 midnight, the Moon clock will have ticked off

$$(24.0\ \text{h})\sqrt{\frac{g_{\text{Moon}}}{g_{\text{Earth}}}} = (24.0\ \text{h})\sqrt{\frac{1.63\ \text{m/s}^2}{9.80\ \text{m/s}^2}} = 9.79\ \text{h} = 9\ \text{h} + (0.79\ \text{h})\left(\frac{60\ \text{min}}{1\ \text{h}}\right) = 9\ \text{h} + 47\ \text{min}$$

and will read $\boxed{9:47\ \text{AM}}$.

13.38 The coat hanger acts as a physical pendulum and its period of oscillation is $T = 2\pi\sqrt{I/mgd}$, where d is the distance from the pivot to the center of mass. Thus, the moment of inertia about the axis perpendicular to the plane of oscillation and passing through the pivot must be

$$I = mgd\left(\frac{T}{2\pi}\right)^2 = (0.238\ \text{kg})(9.80\ \text{m/s}^2)(0.180\ \text{m})\left(\frac{1.25\ \text{s}}{2\pi}\right)^2 = \boxed{1.66\times10^{-2}\ \text{kg}\cdot\text{m}^2}$$

13.39 From $T = 2\pi\sqrt{L/g}$, the length of a pendulum with period T is $L = \dfrac{gT^2}{4\pi^2}$.

(a) On Earth, with $T = 1.0$ s, $T = 1.0$ s, $L = \dfrac{(9.8\ \text{m/s}^2)(1.0\ \text{s})^2}{4\pi^2} = 0.25\ \text{m} = \boxed{25\ \text{cm}}$

(b) If $T = 1.0$ s on Mars, $L = \dfrac{(3.7\ \text{m/s}^2)(1.0\ \text{s})^2}{4\pi^2} = 0.094\ \text{m} = \boxed{9.4\ \text{cm}}$

(c) and (d) The period of an object on a spring is $T = 2\pi\sqrt{m/k}$, which is independent of the local free-fall acceleration. Thus, the same mass will work on Earth and on Mars. This mass is

$$m = \frac{kT^2}{4\pi^2} = \frac{(10\ \text{N/m})(1.0\ \text{s})^2}{4\pi^2} = \boxed{0.25\ \text{kg}}$$

13.40 The apparent free-fall acceleration is the vector sum of the actual free-fall acceleration and the negative of the elevator's acceleration. To see this, consider an object that is hanging from a vertical string in the elevator **and appears to be at rest to the elevator passengers**. These passengers believe the tension in the string is the negative of the object's weight, or $\vec{\mathbf{T}} = -m\vec{\mathbf{g}}_{\text{apparent}}$, where $\vec{\mathbf{g}}_{\text{apparent}}$ is the apparent free-fall acceleration in the elevator.

An observer located outside the elevator applies Newton's second law to this object by writing $\Sigma\vec{\mathbf{F}} = \vec{\mathbf{T}} + m\vec{\mathbf{g}} = m\vec{\mathbf{a}}_e$, where $\vec{\mathbf{a}}_e$ is the acceleration of the elevator and all its contents. Thus,

$$\vec{\mathbf{T}} = m\left(\vec{\mathbf{a}}_e - \vec{\mathbf{g}}\right) = -m\vec{\mathbf{g}}_{\text{apparent}},\ \text{which gives}\ \vec{\mathbf{g}}_{\text{apparent}} = \vec{\mathbf{g}} - \vec{\mathbf{a}}_e.$$

(a) If we choose downward as the positive direction, then $\vec{\mathbf{a}}_e = -5.00\ \text{m/s}^2$ in this case and $\vec{\mathbf{g}}_{\text{apparent}} = (9.80 + 5.00)\ \text{m/s}^2 = +14.8\ \text{m/s}^2$ (downward). The period of the pendulum is

$$T = 2\pi\sqrt{\frac{L}{g_{\text{apparent}}}} = 2\pi\sqrt{\frac{5.00\ \text{m}}{14.8\ \text{m/s}^2}} = \boxed{3.65\ \text{s}}$$

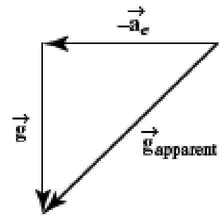
(b) Again choosing downward as positive, $\vec{\mathbf{a}}_e = 5.00\ \text{m/s}^2$ and

$$\vec{\mathbf{g}}_{\text{apparent}} = (9.80 - 5.00)\ \text{m/s}^2 = +4.80\ \text{m/s}^2\ \text{(downward)}$$

in this case. The period is now given by

$$T = 2\pi\sqrt{\frac{L}{g_{\text{apparent}}}} = 2\pi\sqrt{\frac{5.00\ \text{m}}{4.80\ \text{m/s}^2}} = \boxed{6.41\ \text{s}}$$

(c) If $\vec{\mathbf{a}}_e = 5.00\ \text{m/s}^2$ horizontally, the vector sum $\vec{\mathbf{g}}_{\text{apparent}} = \vec{\mathbf{g}} - \vec{\mathbf{a}}_e$



is as shown in the sketch at the right. The magnitude is

$$g_{apparent} = \sqrt{\left(5.00\ \text{m/s}^2\right)^2 + \left(9.80\ \text{m/s}^2\right)^2} = 11.0\ \text{m/s}^2,$$

and the period of the pendulum is

$$T = 2\pi\sqrt{\frac{L}{g_{apparent}}} = 2\pi\sqrt{\frac{5.00\ \text{m}}{11.0\ \text{m/s}^2}} = \boxed{4.24\ \text{s}}$$

13.41 (a) The distance from the bottom of a trough to the top of a crest is twice the amplitude of the wave. Thus, $2A = 8.26$ and $\boxed{A = 4.13\ \text{cm}}$.

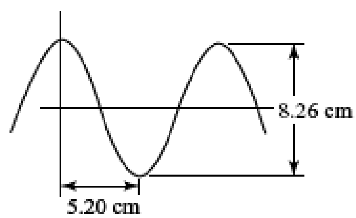


FIGURE P13.41

(b) The horizontal distance from a crest to a trough is a half wavelength. Hence,

$$\lambda/2 = 5.20\ \text{cm} \text{ and } \boxed{\lambda = 10.4\ \text{cm}}$$

(c) The period is

$$T = \frac{1}{f} = \frac{1}{18.0\ \text{s}^{-1}} = \boxed{5.56 \times 10^{-2}\ \text{s}}$$

(d) The wave speed is

$$v = \lambda f = (10.4\ \text{cm})\left(18.0\ \text{s}^{-1}\right) = \boxed{187\ \text{cm/s} = 1.87\ \text{m/s}}$$

13.42 (a) The amplitude is the magnitude of the maximum displacement from equilibrium (at $x = 0$). Thus, $\boxed{A = 2.00\ \text{cm}}$.

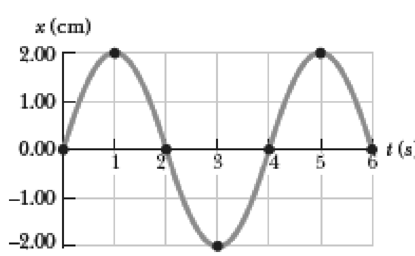


FIGURE P13.42

(b) The period is the time for one full cycle of the motion. Therefore, $\boxed{T = 4.00\ \text{s}}$.

(c) The period may be written as $T = 2\pi/\omega$, so the angular frequency is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{4.00\text{ s}} = \boxed{\frac{\pi}{2}\text{ rad/s}}$$

(d) The total energy may be expressed as $E = \frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$. Thus, $v_{max} = A\sqrt{k/m}$, and

since $\omega = \sqrt{k/m}$, this becomes $v_{max} = \omega A$ and yields

$$v_{max} = \omega A = \left(\frac{\pi}{2}\text{ rad/s}\right)(2.00\text{ cm}) = \boxed{\pi\text{ cm/s}}$$

(e) The spring exerts maximum force, $|F| = k|x|$, when the object is at maximum distance from equilibrium, i.e., at $|x| = A = 2.00\text{ cm}$. Thus, the maximum acceleration of the object is

$$a_{max} = \frac{|F_{max}|}{m} = \frac{kA}{m} = \omega^2 A = \left(\frac{\pi}{2}\text{ rad/s}\right)^2(2.00\text{ cm}) = \boxed{4.93\text{ cm/s}^2}$$

(f) The general equation for position as a function of time for an object undergoing simple harmonic motion with $t = 0$ when $x = 0$ is $x = A\sin(\omega t)$. For this oscillator, this becomes

$$\boxed{x = (2.00\text{ cm})\sin\left(\frac{\pi}{2}t\right)}$$

13.43 (a) The speed of propagation for a wave is the product of its frequency and its wavelength, $v = \lambda f$. Thus, the frequency must be

$$f = \frac{v}{\lambda} = \frac{3.00\times10^8\text{ m/s}}{5.50\times10^{-7}\text{ m}} = \boxed{5.45\times10^{14}\text{ Hz}}$$

(b) The period is $T = \frac{1}{f} = \frac{1}{5.45\times10^{14}\text{ Hz}} = \boxed{1.83\times10^{-15}\text{ s}}$

13.44 (a) The frequency of a transverse wave is the number of crests that pass a given point each second. Thus, if 5.00 crests pass in 14.0 seconds, the frequency is

$$f = \frac{5.00}{14.0\text{ s}} = 0.357\text{ s}^{-1} = \boxed{0.357\text{ Hz}}$$

(b) The wavelength of the wave is the distance between successive maxima or successive minima. Thus, $\lambda = 2.76\text{ m}$ and the wave speed is

$$v = \lambda f = (2.76\text{ m})(0.357\text{ s}^{-1}) = \boxed{0.985\text{ m/s}}$$

13.45 The speed of the wave is

$$v = \frac{\Delta x}{\Delta t} = \frac{425\text{ cm}}{10.0\text{ s}} = 42.5\text{ cm/s}$$

and the frequency is number of vibrations occurring each second, or $f = 40.0\text{ vib}/30.0\text{ s}$.

Thus, $\lambda = \frac{v}{f} = \frac{42.5\text{ cm/s}}{40.0\text{ vib}/30.0\text{ s}} = \frac{(42.5\text{ cm/s})(30.0\text{ s})}{40.0\text{ vib}} = \boxed{31.9\text{ cm}}$

13.46 From $v = \lambda f$, the wavelength (and size of smallest detectable insect) is

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{60.0 \times 10^3 \text{ Hz}} = 5.72 \times 10^{-3} \text{ m} = \boxed{5.72 \text{ mm}}$$

13.47 The frequency of the wave (that is, the number of crests passing the cork each second) is $f = 2.00 \text{ s}^{-1}$ and the wavelength (distance between successive crests) is $\lambda = 8.50 \text{ cm}$. Thus, the wave speed is

$$v = \lambda f = (8.50 \text{ cm})(2.00 \text{ s}^{-1}) = 17.0 \text{ cm/s} = 0.170 \text{ m/s}$$

and the time required for the ripples to travel 10.0 m over the surface of the water is

$$\Delta t = \frac{\Delta x}{v} = \frac{10.0 \text{ m}}{0.170 \text{ m/s}} = \boxed{58.8 \text{ s}}$$

13.48 (a) When the boat is at rest in the water, the speed of the wave relative to the boat is the same as the speed of the wave relative to the water, $v = 4.0 \text{ m/s}$. The frequency detected in this case is

$$f = \frac{v}{\lambda} = \frac{4.0 \text{ m/s}}{20 \text{ m}} = \boxed{0.20 \text{ Hz}}$$

(b) Taking eastward as positive, $\vec{v}_{\text{wave,boat}} = +4.0 \text{ m/s} - (-1.0 \text{ m/s}) = +5.0 \text{ m/s}$ (see the discussion of relative velocity in Chapter 3 of the textbook) gives

$$\vec{v}_{\text{wave,boat}} = +4.0 \text{ m/s} - (-1.0 \text{ m/s}) = +5.0 \text{ m/s} \quad \text{and} \quad v_{\text{boat,wave}} = \left|\vec{v}_{\text{wave,boat}}\right| = 5.0 \text{ m/s}$$

Thus,

$$f = \frac{v_{\text{boat,wave}}}{\lambda} = \frac{5.0 \text{ m/s}}{20 \text{ m}} = \boxed{0.25 \text{ Hz}}$$

13.49 The down and back distance is $4.00 \text{ m} + 4.00 \text{ m} = 8.00 \text{ m}$. The speed is then

$$v = \frac{d_{\text{total}}}{t} = \frac{4(8.00 \text{ m})}{0.800 \text{ s}} = 40.0 \text{ m/s} = \sqrt{F/\mu}$$

Now,

$$\mu = \frac{m}{L} = \frac{0.200 \text{ kg}}{4.00 \text{ m}} = 5.00 \times 10^{-2} \text{ kg/m}$$

so

$$F = \mu v^2 = (5.00 \times 10^{-2} \text{ kg/m})(40.0 \text{ m/s})^2 = \boxed{80.0 \text{ N}}$$

13.50 The speed of the wave is

$$v = \frac{\Delta x}{\Delta t} = \frac{20.0 \text{ m}}{0.800 \text{ s}} = 25.0 \text{ m/s}$$

and the mass per unit length of the rope is $\mu = m/L = 0.350 \text{ kg/m}$. Thus, from $v = \sqrt{F/\mu}$, we obtain

$$F = v^2 \mu = (25.0 \text{ m/s})^2 (0.350 \text{ kg/m}) = \boxed{219 \text{ N}}$$

13.51 $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{1\,350\text{ N}}{5.00\times10^{-3}\text{ kg/m}}} = \boxed{5.20\times10^{2}\text{ m/s}}$

13.52 (a) $f = \dfrac{1}{T} \quad\rightarrow\quad T = \dfrac{1}{f} \quad\rightarrow\quad [T] = \dfrac{1}{[f]} = \dfrac{1}{\text{T}^{-1}} = \text{T} \quad \boxed{\text{units are seconds}}$

$$v = \sqrt{\frac{T}{\mu}} \quad\rightarrow\quad T = \mu v^2 \quad\rightarrow\quad [T] = [\mu][v^2] = \frac{\text{M}}{\text{L}}\cdot\frac{\text{L}^2}{\text{T}^2} = \frac{\text{ML}}{\text{T}^2} \quad \boxed{\text{units are newtons}}$$

(b) $\boxed{\text{The first } T \text{ is period of time; the second is force of tension.}}$

13.53 (a) The mass per unit length is

$$\mu = \frac{m}{L} = \frac{0.0600\text{ kg}}{5.00\text{ m}} = 1.20\times10^{-2}\text{ kg/m}$$

From $v = \sqrt{F/\mu}$, the required tension in the string is

$$F = v^2\mu = (50.0\text{ m/s})^2\left(1.20\times10^{-2}\text{ kg/m}\right) = \boxed{30.0\text{ N}}$$

(b) $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{8.00\text{ N}}{1.20\times10^{-2}\text{ kg/m}}} = \boxed{25.8\text{ m/s}}$

13.54 The mass per unit length of the wire is

$$\mu = \frac{m}{L} = \frac{4.00\times10^{-3}\text{ kg}}{1.60\text{ m}} = 2.50\times10^{-3}\text{ kg/m},$$

and the speed of the pulse is

$$v = \frac{L}{\Delta t} = \frac{1.60\text{ m}}{0.0361\text{ s}} = 44.3\text{ m/s}$$

The tension in the wire is $F = mg = \mu v^2$, so the lunar acceleration of gravity must be

$$g = \frac{v^2\mu}{m} = \frac{(44.3\text{ m/s})^2\left(2.50\times10^{-3}\text{ kg/m}\right)}{3.00\text{ kg}} = \boxed{1.64\text{ m/s}^2}$$

13.55 The period of the pendulum is $T = 2\pi\sqrt{L/g}$, so the length of the string is

$$L = \frac{gT^2}{4\pi^2} = \frac{\left(9.80\text{ m/s}^2\right)(2.00\text{ s})^2}{4\pi^2} = 0.993\text{ m}$$

The mass per unit length of the string is then

$$\mu = \frac{m}{L} = \frac{0.060\,0\text{ kg}}{0.993\text{ m}} = 6.04\times10^{-2}\ \frac{\text{kg}}{\text{m}}$$

When the pendulum is vertical and stationary, the tension in the string is

$$F = M_{\text{ball}}\,g = (5.00\text{ kg})\left(9.80\text{ m/s}^2\right) = 49.0\text{ N}$$

and the speed of transverse waves in it is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{49.0 \text{ N}}{6.04 \times 10^{-2} \text{ kg/m}}} = \boxed{28.5 \text{ m/s}}$$

13.56 If $\mu_1 = m_1/L$ is the mass per unit length for the first string, then $\mu_2 = m_2/L = m_1/2L = \mu_1/2$ is that of the second string. Thus, with $F_2 = F_1 = F$, the speed of waves in the second string is

$$v_2 = \sqrt{\frac{F}{\mu_2}} = \sqrt{\frac{2F}{\mu_1}} = \sqrt{2}\left(\sqrt{\frac{F}{\mu_1}}\right) = \sqrt{2}\, v_1 = \sqrt{2}(5.00 \text{ m/s}) = \boxed{7.07 \text{ m/s}}$$

13.57 (a) The tension in the string is $F = mg = (3.00 \text{ kg})(9.80 \text{ m/s}^2) = 29.4$ N. Then, from $v = \sqrt{F/\mu}$, the mass per unit length is

$$\mu = \frac{F}{v^2} = \frac{29.4 \text{ N}}{(24.0 \text{ m/s})^2} = \boxed{5.10 \times 10^{-2} \text{ kg/m}}$$

(b) When $m = 2.00$ kg, the tension is

$$F = mg = (2.00 \text{ kg})(9.80 \text{ m/s}^2) = 19.6 \text{ N}$$

and the speed of transverse waves in the string is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{19.6 \text{ N}}{5.10 \times 10^{-2} \text{ kg/m}}} = \boxed{19.6 \text{ m/s}}$$

13.58 If the tension in the wire is *F*, the tensile stress is $Stress = F/A$, so the speed of transverse waves in the wire may be written as

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{A \cdot Stress}{m/L}} = \sqrt{\frac{Stress}{m/(A \cdot L)}}$$

But $A \cdot L = V =$ volume, so $m/(A \cdot L) = \rho =$ density. Thus, $v = \sqrt{Stress/\rho}$.

Taking the density of steel to be equal to that of iron, the maximum speed of waves in the wire is

$$v_{max} = \sqrt{\frac{(Stress)_{max}}{\rho_{steel}}} = \sqrt{\frac{2.70 \times 10^9 \text{ Pa}}{7.86 \times 10^3 \text{ kg/m}^3}} = \boxed{586 \text{ m/s}}$$

13.59 (a) The speed of transverse waves in the line is $v = \sqrt{F/\mu}$, with $\mu = m/L$ being the mass per unit length. Therefore,

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{m/L}} = \sqrt{\frac{FL}{m}} = \sqrt{\frac{(12.5 \text{ N})(38.0 \text{ m})}{2.65 \text{ kg}}} = \boxed{13.4 \text{ m/s}}$$

(b) The worker could throw an object, such as a snowball, at one end of the line to set up a pulse, and use a stopwatch to measure the time it takes a pulse to travel the length of the line. From this measurement, the worker would have an estimate of the wave speed, which in turn can be used to estimate the tension.

13.60 (a) In making *n* round trips along the length of the line, the total distance traveled by the pulse is $\Delta x = n(2L) = 2nL$. The wave speed is then

$$v = \frac{\Delta x}{t} = \boxed{\frac{2nL}{t}}$$

(b) From $v = \sqrt{F/\mu}$ as the speed of transverse waves in the line, the tension is

$$F = \mu v^2 = \left(\frac{M}{L}\right)\left(\frac{2nL}{t}\right)^2 = \left(\frac{M}{L}\right)\left(\frac{4n^2L^2}{t^2}\right) = \boxed{\frac{4n^2ML}{t^2}}$$

13.61 (a) $\boxed{\text{Constructive interference}}$ produces the maximum amplitude

$$A'_{max} = A_1 + A_2 = 0.30\text{ m} + 0.20\text{ m} = \boxed{0.50\text{ m}}$$

(b) $\boxed{\text{Destructive interference}}$ produces the minimum amplitude

$$A'_{min} = A_1 - A_2 = 0.30\text{ m} - 0.20\text{ m} = \boxed{0.10\text{ m}}$$

13.62 We are given that $x = A\cos(\omega t) = (0.25\text{ m})\cos(0.4\pi t)$.

(a) By inspection, the amplitude is seen to be $A = \boxed{0.25\text{ m}}$.

(b) The angular frequency is $\omega = 0.4\pi\text{ rad/s}$. But $\omega = \sqrt{k/m}$, so the spring constant is

$$k = m\omega^2 = (0.30\text{ kg})(0.4\pi\text{ rad/s})^2 = \boxed{0.47\text{ N/m}}$$

(c) **Note**: Your calculator must be in *radians mode* for part (c).

At $t = 0.30\text{ s}$, $x = (0.25\text{ m})\cos\left[(0.4\pi\text{ rad/ s})(0.30\text{ s})\right] = \boxed{0.23\text{ m}}$

(d) From conservation of mechanical energy, the speed at displacement x is given by

$v = \omega\sqrt{A^2 - x^2}$. Thus, at $t = 0.30\text{ s}$, when $x = 0.23\text{ m}$, the speed is

$$v = (0.4\pi\text{ rad/s})\sqrt{(0.25\text{ m})^2 - (0.23\text{ m})^2} = \boxed{0.12\text{ m/s}}$$

13.63 (a) The period of a vibrating object-spring system is $T = 2\pi/\omega = 2\pi\sqrt{m/k}$, so the spring constant is

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(2.00\text{ kg})}{(0.600\text{ s})^2} = \boxed{219\text{ N/m}}$$

(b) If $T = 1.05\text{ s}$ for mass m_2, this mass is

$$m_2 = \frac{kT^2}{4\pi^2} = \frac{(219\text{ N/m})(1.05\text{ s})^2}{4\pi^2} = \boxed{6.12\text{ kg}}$$

13.64 (a) The period is the reciprocal of the frequency, or

$$T = \frac{1}{f} = \frac{1}{196\text{ s}^{-1}} = 5.10\times10^{-3}\text{ s} = \boxed{5.10\text{ ms}}$$

(b) $\lambda = \dfrac{v_{sound}}{f} = \dfrac{343\text{ m/s}}{196\text{ s}^{-1}} = \boxed{1.75\text{ m}}$

13.65 (a) The period of a simple pendulum is $T = 2\pi\sqrt{\ell/g}$, so the period of the first system is

$$T_1 = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{0.700 \text{ m}}{9.80 \text{ m/s}^2}} = \boxed{1.68 \text{ s}}$$

(b) The period of an object-spring system is $T = 2\pi\sqrt{m/k}$, so if the period of the second system is $T_2 = T_1$, then $2\pi\sqrt{m/k} = 2\pi\sqrt{\ell/g}$ and the spring constant is

$$k = \frac{mg}{\ell} = \frac{(1.20 \text{ kg})(9.80 \text{ m/s}^2)}{0.700 \text{ m}} = \boxed{16.8 \text{ N/m}}$$

13.66 Since the spring is "light," we neglect any small amount of energy lost in the collision with the spring, and apply conservation of mechanical energy from when the block first starts until it comes to rest again. This gives

$$\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i \text{, or } 0 + 0 + \frac{1}{2}kx_{\max}^2 = 0 + 0 + mgh_i$$

Thus, $$x_{\max} = \sqrt{\frac{2mgh_i}{k}} = \sqrt{\frac{2(0.500 \text{ kg})(9.80 \text{ m/s}^2)(2.00 \text{ m})}{20.0 \text{ N/m}}} = \boxed{0.990 \text{ m}}$$

13.67 Choosing $PE_g = 0$ at the initial height of the 3.00-kg object, conservation of mechanical energy gives $\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i$, or $\frac{1}{2}mv^2 + mg(-x) + \frac{1}{2}kx^2 = 0$, where v is the speed of the object after falling distance x.

(a) When $v = 0$, the non-zero solution to the energy equation from above gives $\frac{1}{2}kx_{\max}^2 = mgx_{\max}$, or

$$k = \frac{2mg}{x_{\max}} = \frac{2(3.00 \text{ kg})(9.80 \text{ m/s}^2)}{0.100 \text{ m}} = \boxed{588 \text{ N/m}}$$

(b) When $x = 5.00 \text{ cm} = 0.050\,0 \text{ m}$, the energy equation gives $v = \sqrt{2gx - kx^2/m}$, or

$$v = \sqrt{2(9.80 \text{ m/s}^2)(0.050\,0 \text{ m}) - \frac{(588 \text{ N/m})(0.050\,0 \text{ m})^2}{3.00 \text{ kg}}} = \boxed{0.700 \text{ m/s}}$$

13.68 (a) We apply conservation of mechanical energy from *just after* the collision until the block comes to rest. Conservation of energy gives $\left(KE + PE_s\right)_f = \left(KE + PE_s\right)_i$ or $0 + \frac{1}{2}kx_f^2 = \frac{1}{2}MV^2 + 0$. The speed of the block just after the collision is then

$$V = \sqrt{\frac{kx_f^2}{M}} = \sqrt{\frac{(900 \text{ N/m})(0.050\,0 \text{ m})^2}{1.00 \text{ kg}}} = 1.50 \text{ m/s}$$

Now, we apply conservation of momentum from just before impact to immediately after the collision. This gives $m\left(v_{\text{bullet}}\right)_i + 0 = m\left(v_{\text{bullet}}\right)_f + MV$, or

$$\left(v_{\text{bullet}}\right)_f = \left(v_{\text{bullet}}\right)_i - \left(\frac{M}{m}\right)V = 400 \text{ m/s} - \left(\frac{1.00 \text{ kg}}{5.00 \times 10^{-3} \text{ kg}}\right)(1.5 \text{ m/s}) = \boxed{100 \text{ m/s}}$$

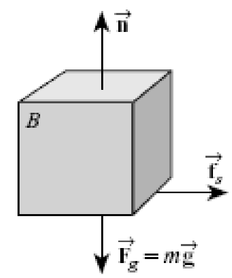
(b) The mechanical energy converted into internal energy during the collision is

$$\Delta E = KE_i - \Sigma KE_f = \tfrac{1}{2}m\left(v_{\text{bullet}}\right)_i^2 - \tfrac{1}{2}m\left(v_{\text{bullet}}\right)_f^2 - \tfrac{1}{2}MV^2, \text{ or}$$

$$\Delta E = \frac{1}{2}\left(5.00\times10^{-3}\text{ kg}\right)\left[\left(400\text{ m/s}\right)^2 - \left(100\text{ m/s}\right)^2\right] - \frac{1}{2}\left(1.00\text{ kg}\right)\left(1.50\text{ m/s}\right)^2$$

$$\Delta E = \boxed{374\text{ J}}$$

13.69 The maximum acceleration of the oscillating system is



$$a_{\max} = \omega^2 A = \left(2\pi f\right)^2 A$$

The friction force, f_s, acting between the two blocks must be capable of accelerating block *B* at this rate. When block *B* is on the verge of slipping, $f_s = \left(f_s\right)_{\max} = \mu_s n = \mu_s mg = ma_{\max}$ and we must have

$$a_{\max} = \left(2\pi f\right)^2 A = \mu_s g$$

Thus, $A = \dfrac{\mu_s g}{\left(2\pi f\right)^2} = \dfrac{(0.600)\left(9.80\text{ m/s}^2\right)}{\left[2\pi(1.50\text{ Hz})\right]^2} = 6.62\times10^{-2}\text{ m} = \boxed{6.62\text{ cm}}$

13.70 (a) When the gun is fired, the energy initially stored as elastic potential energy in the spring is transformed into kinetic energy of the bullet. Assuming no loss of energy, we have $\frac{1}{2}mv^2 = \frac{1}{2}kx_i^2$, or

$$v = x_i\sqrt{\frac{k}{m}} = (0.200\text{ m})\sqrt{\frac{9.80\text{ N/m}}{1.00\times10^{-3}\text{ kg}}} = \boxed{19.8\text{ m/s}}$$

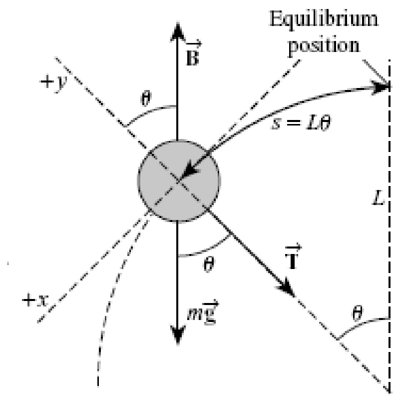
(b) From $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, the time required for the pellet to drop 1.00 m to the floor, starting with $v_{0y} = 0$, is

$$t = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2(-1.00\text{ m})}{-9.80\text{ m/s}^2}} = 0.452\text{ s}$$

The range (horizontal distance traveled during the flight) is then

$$\Delta x = v_{0x}t = (19.8\text{ m/s})(0.452\text{ s}) = \boxed{8.95\text{ m}}$$

13.71 (a) The force diagram at the right shows the forces acting on the balloon when it is displaced distance $s = L\theta$ along the circular arc it follows. The net force tangential to this path is



$$F_{net} = \Sigma F_x = -B\sin\theta + mg\sin\theta = -(B - mg)\sin\theta$$

For small angles, $\sin\theta \approx \theta = s/L$. Also, $mg = (\rho_{He}V)g$ and the buoyant force is $B = (\rho_{air}V)g$. Thus, the net restoring force acting on the balloon is

$$\boxed{F_{net} \approx -\left[\frac{(\rho_{air} - \rho_{He})Vg}{L}\right]s}$$

Observe that this is in the form of Hooke's law, $F = -ks$, with $k = (\rho_{air} - \rho_{He})Vg/L$. Thus, the motion will be $\boxed{\text{simple harmonic}}$.

(b) The period of this simple harmonic motion is given by

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\rho_{He}V}{(\rho_{air} - \rho_{He})Vg/L}} = 2\pi\sqrt{\left(\frac{\rho_{He}}{\rho_{air} - \rho_{He}}\right)\frac{L}{g}}$$

This yields

$$T = 2\pi\sqrt{\left(\frac{0.179\ \text{kg/m}^3}{1.29\ \text{kg/m}^3 - 0.179\ \text{kg/m}^3}\right)\frac{(3.00\ \text{m})}{(9.80\ \text{m/s}^2)}} = \boxed{1.40\ \text{s}}$$

13.72 (a) When the object is given some small upward displacement, the net restoring force exerted on it by the rubber bands is

$$F_{net} = \Sigma F_y = -2F\sin\theta\text{, where } \tan\theta = \frac{y}{L}$$

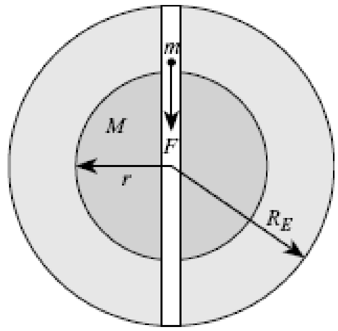
For small displacements, the angle θ will be very small. Then $\sin\theta \approx \tan\theta = y/L$, and the net restoring force is

$$F_{net} = -2F\left(\frac{y}{L}\right) = \boxed{-\left(\frac{2F}{L}\right)y}$$

(b) The net restoring force found in part (a) is in the form of Hooke's law $F = -ky$, with $k = 2F/L$. Thus, the motion will be simple harmonic, and the angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \boxed{\sqrt{\frac{2F}{mL}}}$$

13.73 Newton's law of gravitation is



$$F = -\frac{GMm}{r^2}, \text{ where } M = \rho\left(\frac{4}{3}\pi r^3\right)$$

Thus, $\boxed{F = -\left(\frac{4}{3}\pi\rho G m\right) r}$

which is of Hooke's law form, $F = -kr$, with

$$\boxed{k = \frac{4}{3}\pi\rho G m}$$

13.74 The inner tip of the wing is attached to the end of the spring and always moves with the same speed as the end of the vibrating spring. Thus, its maximum speed is

$$v_{\text{inner, max}} = v_{\text{spring, max}} = A\sqrt{\frac{k}{m}} = (0.20 \text{ cm})\sqrt{\frac{4.7\times10^{-4} \text{ N/m}}{0.30\times10^{-3} \text{ kg}}} = 0.25 \text{ cm/s}$$

Treating the wing as a rigid bar, all points in the wing have the same angular velocity at any instant in time. As the wing rocks on the fulcrum, the inner tip and outer tips follow circular paths of different radii. Since the angular velocities of the tips are always equal, we may write $\omega = \frac{v_{\text{outer}}}{r_{\text{outer}}} = \frac{v_{\text{inner}}}{r_{\text{inner}}}$. The maximum speed of the outer tip is then

$$v_{\text{outer, max}} = \left(\frac{r_{\text{outer}}}{r_{\text{inner}}}\right) v_{\text{inner, max}} = \left(\frac{15.0 \text{ mm}}{3.00 \text{ mm}}\right)(0.25 \text{ cm/s}) = \boxed{1.3 \text{ cm/s}}$$

13.75 (a) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{500 \text{ N/m}}{2.00 \text{ kg}}} = \boxed{15.8 \text{ rad/s}}$

(b) Apply Newton's second law to the block while the elevator is accelerating:

$$\Sigma F_y = F_s - mg = ma_y$$

With $F_s = kx$ and $a_y = g/3$, this gives $kx = m(g + g/3)$, or

$$x = \frac{4mg}{3k} = \frac{4(2.00 \text{ kg})(9.80 \text{ m/s}^2)}{3(500 \text{ N/m})} = 5.23 \times 10^{-2} \text{ m} = \boxed{5.23 \text{ cm}}$$

13.76 (a) Note that as the spring passes through the vertical position, the object is moving in a circular arc of radius $L - y_f$. Also, observe that the *y*-coordinate of the object at this point must be negative $(y_f < 0)$, so the spring is stretched and exerting an upward tension force of magnitude greater than the object's weight. This is necessary so the object experiences a net force toward the pivot to supply the needed centripetal acceleration in this position. This is summarized by Newton's second law applied to the object at this point, stating

$$\boxed{\Sigma F_y = -ky_f - mg = \frac{mv^2}{L - y_f}}$$

(b) Conservation of energy requires that $E = KE_i + PE_{g,i} + PE_{s,i} = KE_f + PE_{g,f} + PE_{s,f}$, or

$$E = 0 + mgL + 0 = \frac{1}{2}mv^2 + mgy_f + \frac{1}{2}ky_f^2$$

reducing to $\boxed{mv^2 = 2mg(L - y_f) - ky_f^2}$

(c) From the result of part (a), observe that

$$mv^2 = -(L - y_f)(ky_f + mg)$$

Substituting this into the result from part (b) gives

$$2mg(L - y_f) = -(L - y_f)(ky_f + mg) + ky_f^2$$

After expanding and regrouping terms, this becomes

$$(2k)y_f^2 + (3mg - kL)y_f + (-3mgL) = 0$$

which is a quadratic equation $ay_f^2 + by_f + c = 0$, with

$$a = 2k = 2(1\,250 \text{ N/m}) = 2.50 \times 10^3 \text{ N/m}$$

$$b = 3mg - kL = 3(5.00 \text{ kg})(9.80 \text{ m/s}^2) - (1\,250 \text{ N/m})(1.50 \text{ m}) = -1.73 \times 10^3 \text{ N}$$

and $c = -3mgL = -3(5.00 \text{ kg})(9.80 \text{ m/s}^2)(1.50 \text{ m}) = -221 \text{ N} \cdot \text{m}$

Applying the quadratic formula, keeping only the negative solution [see the discussion in part (a)] gives

$$y_f = \frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{-\left(-1.73\times10^3\right)-\sqrt{\left(-1.73\times10^3\right)^2-4\left(2.50\times10^3\right)\left(-221\right)}}{2\left(2.50\times10^3\right)}$$

or $\boxed{y_f = -0.110\text{ m}}$

(d) Because the length of this pendulum varies and is longer throughout its motion than a simple pendulum of length L, $\boxed{\text{its period will be greater than}}$ that of a simple pendulum.